Field representations in composite chiral-ferrite media by cylindrical vector wave functions

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Composite chiral-ferrite media, with the introduction of anisotropy into the well-studied chiral materials in order to blend the effects of Faraday rotation and optical activity, have potential applications in chirality management. In the present consideration, based on the concept of characteristic waves and the method of angular spectral expansion, field representations in this class of media are developed. The analysis reveals that the solutions of the source-free Maxwell's equation for homogeneous composite chiral-ferrite media can be represented in sum-integral forms of the circular cylindrical vector wave functions in isotropic media. The addition theorem of vector wave functions for composite chiral-ferrite media can be derived from that of vector wave functions for isotropic media. An application of the present theory in scattering is presented to show how to use these formulations in a practical way.

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I. INTRODUCTION

The vector wave functions, which are important concepts in electromagnetism, have found versatile applications and presented great advantages compared with other methods (e.g., three-dimensional moment method [1], coupled-dipole method [2], and integral-equation technique [3]). For instance, the circular cylindrical vector wave functions have been successfully used in studying the scattering properties of a multilayer chiral cylinder [4], and the radiation characteristics of a dipole antenna in the proximity of a gyroelectric cylinder [5]. Unfortunately, to the best knowledge of the author, only in biisotropic media [6] (with isotropic media and the well-studied reciprocal chiral media as their subset cases) and gyrotropic media [5], have electromagnetic waves been represented in terms of the vector wave functions. Therefore, field representations in complex media need to be developed, so as to provide methodological convenience in investigating the electromagnetic properties of these materials.

With the advancement of polymer synthesis techniques, increasing attention has been attracted to the area of interaction between electromagnetic waves and chiral media, in order to determine how to use these materials to provide better solutions to current engineering problems. It has been discovered that chiral materials can be utilized to construct antireflection coatings, reciprocal microwave components, and antenna radomes, whose physical behaviors are determined by the chirality degree. However, only limited methods exist in the control of chirality degree once chiral materials are fabricated, except with the introduction of certain forms of anisotropy. With chirality management as investigation motivation, Engheta, Jaggard, and Kowarz [7] investigated the propagation characteristics of electromagnetic waves in unbounded Faraday chiral media, which blend the effects of Faraday rotation with those of optical activity. Lately, the reflection and transmission properties of electromagnetic waves in Faraday chiral slab with finite longitudinal extent were researched and the interplay of Faraday and chiral effects was studied [8]. Most recently, general formulations of composite chiral-ferrite media, such as nonreciprocal properties, dyadic Green's functions in unbounded space, dispersion relations, and polarization characteristics, are developed by Krowne [9]. Nevertheless, much effort is still needed in order to achieve a throughout understanding and exploiting of the chirality management.

A composite chiral-ferrite medium, formed by immersing chiral objects in a magnetized ferrite, is a subset of the wider class referred to as bianisotropic media. Excellent works in general bianisotropic media have been done by Post [10], Kong [11], and Chen [12] among others. In contradistinction to these general considerations, the present contribution is aimed to develop the field representations in composite chiral-ferrite media in terms of the cylindrical vector wave functions. The formulations lead to compact and explicit expressions of the field representations in this class of media. In addition, to make the efficient recursive algorithm developed by Chew [13] available to multiscatterers and multilayered structures consisted of composite chiral-ferrite media, an outline to derive the addition theorem is described. As an illustrative example, electromagnetic scattering of a normally incident plane wave by an infinitely long composite chiral-ferrite rod is studied.

In this manuscript, the harmonic $\exp(i\omega t)$ time dependence is assumed and suppressed throughout.

II. THEORETICAL DEVELOPMENT

From a phenomenological point of view, a composite chiral-ferrite medium can be characterized by a set of constitutive relations [9]

$$\mathbf{D}=\varepsilon\mathbf{E}+\xi_{c1}\mathbf{H}\ , \tag{1a}$$

$$\mathbf{B}=\underline{\mu}\mathbf{H}+\xi_{c2}\mathbf{E}\ , \tag{1b}$$

where

$$\underline{\mu}=\begin{bmatrix}\mu & ik_m & 0\\ -ik_m & \mu & 0\\ 0 & 0 & \mu_0\end{bmatrix} \tag{1c}$$

is the modified permeability tensor of a magnetically biased ferrite, taking into the contributions due to chirality. (ξ_{c1}, ξ_{c2}) and ε are the chirality parameters and permittivity, respectively.

Substituting the constitutive relations (1a) and (1b) into the source-free Maxwell's equations, the **H**-field vector wave equation in this composite medium is yielded [9]

$$\nabla\times\nabla\times\mathbf{H}+i\omega(\xi_{c2}-\xi_{c1})\nabla\times\mathbf{H}-\omega^2\varepsilon\mu_0[\underline{\mu}/\mu_0-\xi_{c1}\xi_{c2}\overleftrightarrow{\mathrm{I}}/(\mu_0\varepsilon)]\mathbf{H}=\mathbf{0}\ , \tag{2}$$

with $\overleftrightarrow{\mathrm{I}}$ denoting the unit dyadic.

The characteristic waves corresponding to (2) can be examined in the Fourier domain by introducing the three-dimensional transformation

$$\mathbf{H}(\mathbf{r})=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{H}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{r}}dk_x dk_y dk_z\ , \tag{3}$$

where $\mathbf{k}=k_x\hat{\mathbf{e}}_x+k_y\hat{\mathbf{e}}_y+k_z\hat{\mathbf{e}}_z$, and $\hat{\mathbf{e}}_j$ indicates unit vector in the j direction. Substituting Eq. (3) into (2) and after some proper vector algebraic manipulation, we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\underline{W}\cdot\mathbf{H}(\mathbf{k})dk_x dk_y dk_z=\mathbf{0}\ , \tag{4}$$

where the spectral matrix $\underline{W}$ is determined as

$$\underline{W}=\begin{bmatrix}k^2-k_x^2-a & -k_xk_y-bk_z-ic & -k_xk_z+bk_y\\ -k_xk_y+bk_z+ic & k^2-k_y^2-a & -k_yk_z-bk_x\\ -k_xk_z-bk_y & -k_yk_z+bk_x & k^2-k_z^2-a'\end{bmatrix}, \tag{5}$$

with the notations

$$\begin{aligned}&a=\omega^2\varepsilon\mu[1-\xi_{c1}\xi_{c2}/(\varepsilon\mu)]\ ,\\ &b=\omega(\xi_{c2}-\xi_{c1})\ ,\\ &c=\omega^2\varepsilon k_m\ ,\\ &a'=\omega^2\varepsilon\mu_0[1-\xi_{c1}\xi_{c2}/(\varepsilon\mu_0)]\ ,\end{aligned} \tag{6}$$

and $k^2=k_x^2+k_y^2+k_z^2$. For nontrivial solutions of (4), the determinant of matrix $\underline{W}$ operating on $\mathbf{H}(\mathbf{k})$ must be equal to zero. Straight-forward algebraic manipulation leads to the characteristic equation

$$ak_\rho^4+k_\rho^2[(k_z^2-a)(a+a')+c^2+ab^2]+[(k_z^2-a)^2-(bik_z-c)^2]a'=0\ , \tag{7}$$

where $k_\rho^2=k_x^2+k_y^2$.

In the following analysis, the roots of (7) are designated as $k_\rho=k_{\rho q}$ $(q=1,2,3,4)$, which are functions of k_z. Their corresponding eigenvectors, expressed in a circular cylindrical coordinate system, can be easily obtained from Eq. (4) in conjunction with the rectangular-circular cylindrical coordinate transformation

$$\begin{aligned}\mathbf{H}_q(k_z,\phi_k)=&[A_q(k_z)\cos(\phi-\phi_k)+B_q(k_z)\sin(\phi-\phi_k)]\hat{e}_\rho\\ &+[-A_q(k_z)\sin(\phi-\phi_k)\\ &+B_q(k_z)\cos(\phi-\phi_k)]\hat{e}_\phi+\hat{e}_z\ ,\end{aligned} \tag{8}$$

where the spectral parameters are

$$A_q(k_z)=k_{\rho q}[k_z(k_{\rho q}^2+k_z^2-a+b^2)+ibc]/D_q(k_z)\ , \tag{9a}$$

$$B_q(k_z)=ik_{\rho q}(iab-ck_z)/D_q(k_z)\ , \tag{9b}$$

$$D_q(k_z)=(k_z^2-a)(k_{\rho q}^2+k_z^2-a)-(ibk_z-c)^2\ , \tag{9c}$$

and $\phi_k=\tan^{-1}(k_y/k_x)$, $\phi=\tan^{-1}(y/x)$.

Returning to (3) and noting $k_{\rho 3}=-k_{\rho 1}$, $k_{\rho 4}=-k_{\rho 2}$, we can express the magnetic field in circular cylinderical coordinate system in terms of the above mentioned eigenvectors

$$\mathbf{H}(\mathbf{r})=\sum_{q=1}^{2}\int_{-\infty}^{\infty}dk_z\int_{\phi_k=0}^{2\pi}d\phi_k e^{-i[k_zz+k_{\rho q}\rho\cos(\phi-\phi_k)]}\times\mathbf{H}_q(k_z,\phi_k)H_{qz}(k_z,\phi_k)\ , \tag{10}$$

where $\rho=(x^2+y^2)^{1/2}$, and $H_{qz}(k_z,\phi_k)$ is the amplitude of the spectral longitudinal component of the magnetic field. The symmetric roots $k_{\rho 3}$ and $k_{\rho 4}$ are not included in the summations of (10), since these symmetric roots are automatically taken into account as the spectral azimuthal angle ϕ_k spans from 0 to 2π.

Substituting the explicit expression of the eigenvector (8) into (10), the solution $\mathbf{H}(\mathbf{r})$ of the source-free vector wave equation (2) for an infinite composite chiral-ferrite medium is expressed in terms of the scalar cylindrical wave functions. However, in order to have a tractable solutions for the boundary value problems involving cylindrical structures of composite chiral-ferrite media, it is required to transform the expression of (10) into a form resembling the vector wave solution for an isotropic medium. To this end, applying the angular spectral expansion for $H_{qz}(k_z,\phi_k)$ component under the hypothesis that $H_{qz}(k_z,\phi_k)$ is continuous with respect to the ϕ_k variable

$$H_{qz}(k_z,\phi_k)=\sum_{n=-\infty}^{\infty}h_{qn}(k_z)e^{-in\phi_k}\ , \tag{11}$$

we have

$$\mathbf{H}(\mathbf{r})=\sum_{q=1}^{2}\int_{-\infty}^{\infty}dk_z\sum_{n=-\infty}^{\infty}h_{qn}(k_z)\mathbf{H}_{qn}(k_z)\ , \quad (12)$$

where

$$\mathbf{H}_{qn}(k_z)=\int_{\phi_k=0}^{2\pi}d\phi_k e^{-i[k_z z+k_{\rho q}\rho\cos(\phi-\phi_k)+n\phi_k]}\times\mathbf{H}_q(k_z,\phi_k)\ . \quad (13)$$

Substituting into (13) the well-known identity

$$e^{-ik_{\rho q}\rho\cos(\phi-\phi_k)}=\sum_{m=-\infty}^{\infty}(-i)^m J_m(k_{\rho q}\rho)e^{-im(\phi-\phi_k)}\ , \quad (14)$$

and its derivatives with respect to ρ and ϕ, after cumbersome mathematical manipulation by grouping properly the terms involving in the integration for the ϕ_k variable and introducing the circular cylindrical vector wave functions, we end up with (see Appendix for detail)

$$\mathbf{H}(\mathbf{r})=\pi\sum_{q=1}^{2}\int_{-\infty}^{\infty}dk_z\sum_{n=-\infty}^{\infty}(-i)^n h_{qn}(k_z)\times[A_q^h(k_z)\mathbf{M}_n^{(1)}(k_z,k_{\rho q})+B_q^h(k_z)\mathbf{N}_n^{(1)}(k_z,k_{\rho q})+C_q^h(k_z)\mathbf{L}_n^{(1)}(k_z,k_{\rho q})]\ , \quad (15)$$

where the vector wave functions are defined as

$$\mathbf{M}_n^{(j)}(k_z,k_{\rho q})=\left[-\frac{in}{\rho}Z_n^{(j)}(k_{\rho q}\rho)\hat{\mathbf{e}}_\rho-\frac{\partial Z_n^{(j)}(k_{\rho q}\rho)}{\partial\rho}\hat{\mathbf{e}}_\phi\right]e^{-i(n\phi+k_z z)}\ , \quad (16a)$$

$$\mathbf{N}_n^{(j)}(k_z,k_{\rho q})=\frac{1}{k_q}\left[-ik_z\frac{\partial Z_n^{(j)}(k_{\rho q}\rho)}{\partial\rho}\hat{\mathbf{e}}_\rho-\frac{nk_z}{\rho}Z_n^{(j)}(k_{\rho q}\rho)\hat{\mathbf{e}}_\phi+k_{\rho q}^2 Z_n^{(j)}(k_{\rho q}\rho)\hat{\mathbf{e}}_z\right]e^{-i(n\phi+k_z z)}\ , \quad (16b)$$

$$\mathbf{L}_n^{(j)}(k_z,k_{\rho q})=\left[\frac{\partial Z_n^{(j)}(k_{\rho q}\rho)}{\partial\rho}\hat{\mathbf{e}}_\rho-\frac{in}{\rho}Z_n^{(j)}(k_{\rho q}\rho)\hat{\mathbf{e}}_\phi-ik_z Z_n^{(j)}(k_{\rho q}\rho)\hat{\mathbf{e}}_z\right]e^{-i(n\phi+k_z z)}\ , \quad (16c)$$

with

$$Z_n^{(j)}(k_{\rho q}\rho)=\begin{cases}J_n(k_{\rho q}\rho)\ , & j=1\\ Y_n(k_{\rho q}\rho)\ , & j=2\\ H_n^{(1)}(k_{\rho q}\rho)\ , & j=3\\ H_n^{(2)}(k_{\rho q}\rho)\ , & j=4\ ,\end{cases} \quad (16d)$$

and $k_q=(k_{\rho q}^2+k_z^2)^{1/2}$. In Eq. (15), the weighted coefficients of the vector wave functions are found to be

$$A_q^h(k_z)=-2iB_q(k_z)/k_{\rho q}\ , \quad (17a)$$

$$B_q^h(k_z)=-2k_q A_q(k_z)/(k_{\rho q}k_z)+2[1+k_{\rho q}A_q(k_z)/k_z]/k_q\ , \quad (17b)$$

$$C_q^h(k_z)=2ik_z[1+k_{\rho q}A_q(k_z)/k_z]/k_q^2\ . \quad (17c)$$

The representation for the electric field can be easily obtained from the relation [9]

$$\mathbf{E}=-\frac{1}{\varepsilon}\left[\frac{i}{\omega}(\nabla\times\mathbf{H})+\xi_{c1}\mathbf{H}\right], \quad (18)$$

which results in

$$\mathbf{E}(\mathbf{r})=\pi\sum_{q=l}^{2}\int_{-\infty}^{\infty}dk_z\sum_{n=-\infty}^{\infty}(-i)^n h_{qn}(k_z)[A_q^e(k_z)\mathbf{M}_n^{(1)}(k_z,k_{\rho q})+B_q^e(k_z)\mathbf{N}_n^{(1)}(k_z,k_{\rho q})+C_q^e(k_z)\mathbf{L}_n^{(1)}(k_z,k_{\rho q})]\ , \quad (19)$$

with the weighted coefficients of the vector wave functions determined as

$$A_q^e(k_z)=-ik_q B_q^h(k_z)/(\omega\varepsilon)-\xi_{c1}A_q^h(k_z)/\varepsilon\ , \quad (20a)$$

$$B_q^e(k_z)=-ik_q A_q^h(k_z)/(\omega\varepsilon)-\xi_{c1}B_q^h(k_z)/\varepsilon\ , \quad (20b)$$

and

$$C_q^e(k_z)=-\xi_{c1}C_q^h(k_z)/\varepsilon\ . \quad (20c)$$

Since Bessel, Neumann, and Hankel functions of the same order satisfy the identical differential equation, the first kind of vector wave functions in (15) and (19) can be generalized to three other ones, corresponding to Neumann and Hankel functions.

The resulting equations (15) and (19) manifest the fact that the solutions of the source-free Maxwell equations for composite chiral-ferrite media, which are expanded in terms of the circular cylindrical vector wave functions in isotropic media, are superpositions of two transversal waves (*TE* for **M** and *TM* for **N**) and a longitudinal wave.

It should be pointed out that the formulations developed here unify those of gyromagnetic media, and thereby can be theoretically verified by comparing their special forms (for $\xi_{c1}=\xi_2=0$) with the duality forms of the already existed results corresponding to gyroelectric media [5].

Straightforwardly, the addition theorem of circular cylindrical vector wave functions for composite chiral-ferrite media can be directly obtained by using the counterpart of isotropic media [13] in Eqs. (15) and (19).

III. AN APPLICATION EXAMPLE

To illustrate how to use the present theory in a practical way, we study the electromagnetic scattering by an infinitely long composite chiral-ferrite rod due to a normally incident plane wave.

Let us fix the coordinate system so that the scatterer is bounded by the surface $\rho=\rho_0$, and its axis is coincident with the z axis. The surrounding medium is taken to be free space with permittivity ε_0 and permeability μ_0.

In the case of a TM_z polarized incident plane wave illuminating along the $+x$ axis, the incident electromagnetic fields can be expanded in terms of the circular cylindrical vector wave functions [4,14] (note that the circular cylindrical vector wave functions used here are slightly different from those defined in Ref. [4]).

$$\mathbf{E}_{\rm inc}(\mathbf{r})=\hat{\mathbf{e}}_z E_0 e^{-ik_0x} =\int_{-\infty}^{\infty} dk_z \sum_{n=-\infty}^{\infty} (-i)^n E_0 \delta(k_z) \mathbf{N}_n^{(1)}(k_z,k_\rho)\Big/ k_0 , \tag{21a}$$

$$\mathbf{H}_{\rm inc}(\mathbf{r})=-\hat{\mathbf{e}}_y E_0 e^{-ik_0x}/\eta_0 =\int_{-\infty}^{\infty} dk_z \sum_{n=-\infty}^{\infty} (-i)^{n-1} E_0 \delta(k_z) \mathbf{M}_n^{(1)}(k_z,k_\rho)\Big/ (k_0\eta_0) , \tag{21b}$$

where $k_\rho=(k_0^2-k_z^2)^{1/2}$, and $\delta(\,)$ is the Dirac δ function. Here, $k_0=\omega(\varepsilon_0\mu_0)^{1/2}$ and $\eta_0=(\mu_0/\varepsilon_0)^{1/2}$ represent the wave number and wave impedance of free space, respectively. The electromagnetic fields induced in this scatterer can be expressed in terms of the circular cylindrical vector wave functions in the way we have presented in the previous section. And the scattered electromagnetic waves may have TM_z and TE_z components and should be expanded as [4,14]

$$\mathbf{E}_{\rm sca}(\mathbf{r})=\int_{-\infty}^{\infty} dk_z \sum_{n=-\infty}^{\infty} (-i)^n [a_n \mathbf{M}_n^{(4)}(k_z,k_\rho) + b_n \mathbf{N}_n^{(4)}(k_z,k_\rho)] , \tag{22a}$$

$$\mathbf{H}_{\rm sca}(\mathbf{r})=\int_{-\infty}^{\infty} dk_z \sum_{n=-\infty}^{\infty} [(-i)^{n-1}/\eta_0] \times [a_n \mathbf{N}_n^{(4)}(k_z,k_\rho) + b_n \mathbf{M}_n^{(4)}(k_z,k_\rho)] . \tag{22b}$$

In the expressions (21) and (22), the functions $\mathbf{M}_n^{(j)}(k_z,k_\rho)$ and $\mathbf{N}_n^{(j)}(k_z,k_\rho)$ $(j=1,4)$ are the circular cylindrical vector wave functions defined in the previous section.

Applying the widely-employed mode matching method [4,11,14] to have the boundary conditions of continuous tangential electric and magnetic fields satisfied at the outer surface of the scatterer $\rho=\rho_0$, we derive the expansion coefficients of the scattered fields

$$a_n=-\frac{2E_0}{\pi\eta_0\rho_0\Delta_n(\rho_0)}\delta(k_z)[V_{1n}^e(\rho_0)X_{2n}^h(\rho_0) - V_{2n}^e(\rho_0)X_{1n}^h(\rho_0)] , \tag{23a}$$

and

$$b_n=\frac{E_0}{\Delta_n(\rho_0)}\delta(k_z)\left[J_n(k_0\rho_0)C_n(\rho_0) + \frac{i}{\eta_0}J_n'(k_0\rho_0)D_n(\rho_0)\right] , \tag{23b}$$

where

$$V_{qn}^p(\rho_0)=A_q^p(k_z)k_{\rho q}J_n'(k_{\rho q}\rho_0) + \frac{in}{\rho_0}C_q^p(k_z)J_n(k_{\rho q}\rho_0) , \tag{24a}$$

$$X_{qn}^p(\rho_0)=B_q^p(k_z)k_{\rho q}J_n(k_{\rho q}\rho_0) , \tag{24b}$$

for $q=1$, 2, and $p=e,h$; and the primes over the Bessel functions denote derivatives with respect to the argument. In Eqs. (23a) and (23b),

$$C_n(\rho_0)=-k_0H_n^{(2)\prime}(k_0\rho_0) \times[V_{1n}^h(\rho_0)X_{2n}^h(\rho_0)-V_{2n}^h(\rho_0)X_{1n}^h(\rho_0)] -i\omega\varepsilon_0H_n^{(2)}[V_{1n}^e(\rho_0)V_{2n}^h(\rho_0) - V_{2n}^e(\rho_0)V_{1n}^h(\rho_0)] , \tag{25a}$$

$$D_n(\rho_0)=k_0H_n^{(2)\prime}(k_0\rho_0)[X_{1n}^e(\rho_0)X_{2n}^h(\rho_0) -X_{2n}^e(\rho_0)X_{1n}^h(\rho_0)] +i\omega\varepsilon_0H_n^{(2)}[V_{1n}^e(\rho_0)X_{2n}^e(\rho_0) - V_{2n}^e(\rho_0)X_{1n}^e(\rho_0)] , \tag{25b}$$

$$\Delta_n(\rho_0)=-k_0H_n^{(2)}(k_0\rho_0)C_n(\rho_0) -i\omega\varepsilon_0H_n^{(2)\prime}(k_0\rho_0)D_n(\rho_0) . \tag{25c}$$

Since the emergence of Dirac function $\delta(k_z)$ in (23a) and (23b), the infinite integration of the k_z variable for the scattered fields is actually disappeared.

The bistatic echo width, which represents the density of the power scattered by the cylindrical object, is defined as

$$A_\sigma(\phi)=\lim_{\rho\to\infty}2\pi\rho\frac{{\rm Re}[\mathbf{E}_{\rm sca}(\mathbf{r})\times\mathbf{H}_{\rm sca}^*(\mathbf{r})]\cdot\hat{\mathbf{e}}_\rho}{{\rm Re}[\mathbf{E}_{\rm inc}(\mathbf{r})\times\mathbf{H}_{\rm inc}^*(\mathbf{r})]\cdot\hat{\mathbf{e}}_\rho} , \tag{26}$$

where the asterisk indicates complex conjugate, and Re[ ] denotes the real part of the complex function.

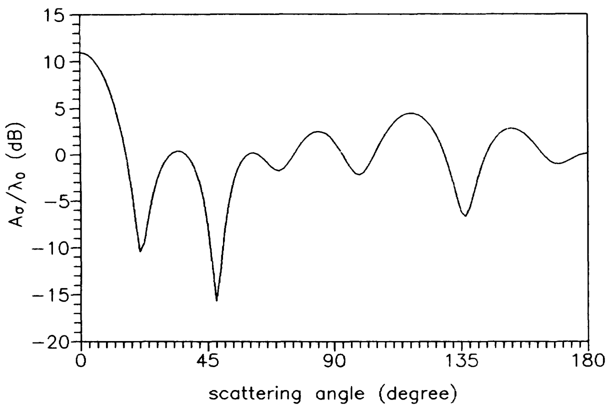

FIG. 1. Scattering pattern of an infinitely long composite chiral-ferrite rod due to a normally incident TM_z polarized plane wave. The scatterer has a radius of $\rho_0 = 1.3\lambda_0$, and constitutive parameters of $\varepsilon = 3.6\varepsilon_0$, $\mu = 2.4\mu_0$, $k_m = 0.15\mu_0$, $\xi_{c1} = 1.131i(\mu_0\varepsilon_0)^{1/2}$, and $\xi_{c2} = 3.016i(\mu_0\varepsilon_0)^{1/2}$.

Recalling the asymptotic expression of the Hankel function in the far region

$$H_n^{(2)}(k_0\rho) = \left[\frac{2}{\pi k_0 \rho}\right]^{1/2} e^{-i[k_0\rho - (2n+1)\pi/4]}, \quad \rho \to \infty , \tag{27}$$

we can rewrite the expression (26) in a more explicit way

$$A_\sigma(\phi) = \frac{4}{k_0 E_0^2}\left[\left|\sum_{n=0}^{\infty}(-1)^n \delta_n a_n \cos(n\phi)\right|^2 + \left|\sum_{n=0}^{\infty}(-1)^n \delta_n b_n \cos(n\phi)\right|^2\right], \tag{28}$$

where δ_n is the Neumann factor, i.e., $\delta_n = 1$ for $n=0$ and 2 for $n>0$.

It should be mentioned that previous to the actual computation, a convergence test was made to check the validity of the numerical results. Figure 1 pictures the scattering pattern of the bistatic echo width, where λ_0 is the wavelength of the incident wave. In this case, the scatterer has a radius of $\rho_0 = 1.3\lambda_0$, and constitutive parameters of $\varepsilon = 3.6\varepsilon_0$, $\mu = 2.4\mu_0$, $k_m = 0.15\mu_0$, $\xi_{c1} = 1.131i(\mu_0\varepsilon_0)^{1/2}$ and $\xi_{c2} = 3.016i(\mu_0\varepsilon_0)^{1/2}$. Since the incident wave is illuminated along the $\phi = 180°$ line, the pattern has a maximum in the forward direction, i.e., $\phi = 0°$.

IV. CONCLUDING REMARKS

In the present contribution, field representations in composite chiral-ferrite media are developed in terms of the circular cylindrical vector wave functions in isotropic media. The formulation is greatly facilitated by using the method of angular spectral expression for the spectral expansion coefficient of the magnetic field. The theory developed here generalizes the canonical solutions of vector wave functions for isotropic media, and recovers the cases of gyromagnetic and biisotropic media. Even if the notations introduced here are somewhat cumbersome which are inevitable due to the complexity of the media we have tried to tackle, the formulations proposed here can be theoretically examined by comparing their special forms with the duality forms of the already existed counterparts. Moreover, it is of interest to note the cylindrical vector wave functions can be expanded as discrete sums of the spherical vector wave functions [15], therefore the theory presented here could be extended to solve the problems of spherical structures. It is believed that the present contribution would be helpful in simplifying the analytical and numerical solutions to boundary value problems of layered structures as well as multiscatterers consisting of composite chiral-ferrite media. An application of the present theory in investigating the electromagnetic scattering by an infinitely long composite chiral-ferrite rod is given, which illustrates the applicability of the proposed theory.

APPENDIX: DERIVATION OF EQUATIONS (15) AND (17)

Substituting the expressions (8) and (13) in (12), we have the magnetic field expanded in terms of eigenvectors in the circular cylindrical coordinate system

$$\mathbf{H}(\mathbf{r}) = \sum_{q=1}^{2}\int_{-\infty}^{\infty} dk_z \sum_{n=-\infty}^{\infty} h_{qn}(k_z)\int_{\phi_k=0}^{2\pi} d\phi_k\, e^{-i[k_z z + k_{\rho q}\rho\cos(\phi-\phi_k) + n\phi_k]} \times \{[A_q(k_z)\cos(\phi-\phi_k) + B_q(k_z)\sin(\phi-\phi_k)]\hat{\mathbf{e}}_\rho + [-A_q(k_z)\sin(\phi-\phi_k) + B_q(k_z)\cos(\phi-\phi_k)]\hat{\mathbf{e}}_\phi + \hat{\mathbf{e}}_z\} . \tag{A1}$$

Then, taking the derivatives of (14) with respect to ρ and ϕ, respectively, we have

$$\cos(\phi-\phi_k)e^{-ik_{\rho q}\rho\cos(\phi-\phi_k)} = \sum_{m=-\infty}^{\infty}(-1)^{m-1}\frac{\partial J_m(k_{\rho q}\rho)}{k_{\rho q}\partial_\rho}e^{-im(\phi-\phi_k)} \tag{A2a}$$

and

$$\sin(\phi-\phi_k)e^{-ik_{\rho q}\rho\cos(\phi-\phi_k)} = -\sum_{m=-\infty}^{\infty}(-1)^m\frac{mJ_m(k_{\rho q}\rho)}{k_{\rho q}\rho}e^{-im(\phi-\phi_k)} . \tag{A2b}$$

Inserting (A2a), (A2b), and (14) into Eq. (A1), we obtain

$$\mathbf{H}(\mathbf{r})=\sum_{q=1}^{2}\int_{-\infty}^{\infty}dk_z\sum_{n=-\infty}^{\infty}h_{qn}(k_z)[P_n(k_z)\hat{\mathbf{e}}_\rho+Q_n(k_z)\hat{\mathbf{e}}_\phi+R_n(k_z)\hat{\mathbf{e}}_z]\ , \tag{A3}$$

where

$$P_n(k_z)=\int_{\phi_k=0}^{2\pi}d\phi_k\sum_{m=-\infty}^{\infty}\left[(-i)^{m-1}\frac{\partial J_m(k_{\rho q}\rho)}{k_{\rho q}\partial\rho}A_q(k_z)-(-i)^m\frac{mJ_m(k_{\rho q}\rho)}{k_{\rho q}\rho}B_q(k_z)\right]e^{-i(k_z z+m\phi)+i(m-n)\phi_k}$$

$$=2\pi(-i)^n\left[\frac{i\partial J_n(k_{\rho q}\rho)A_q(k_z)}{k_{\rho q}\partial\rho}-\frac{nJ_n(k_{\rho q}\rho)B_q(k_z)}{k_{\rho q}\rho}\right]e^{-i(k_z z+n\phi)}\ , \tag{A4}$$

$$Q_n(k_z)=\int_{\phi_k=0}^{2\pi}d\phi_k\sum_{m=-\infty}^{\infty}\left[(-i)^{m-1}\frac{\partial J_m(k_{\rho q}\rho)}{k_{\rho q}\partial\rho}B_q(k_z)+(-i)^m\frac{mJ_m(k_{\rho q}\rho)}{k_{\rho q}\rho}A_q(k_z)\right]e^{-i(k_z z+m\phi)+i(m-n)\phi_k}$$

$$=2\pi(-i)^n\left[\frac{i\partial J_n(k_{\rho q}\rho)B_q(k_z)}{k_{\rho q}\partial\rho}+\frac{nJ_n(k_{\rho q}\rho)A_q(k_z)}{k_{\rho q}\rho}\right]e^{-i(k_z z+n\phi)}\ , \tag{A5}$$

and

$$R_n(k_z)=\int_{\phi_k=0}^{2\pi}d\phi_k\sum_{m=-\infty}^{\infty}[(-i)^m J_m(k_{\rho q}\rho)]e^{-i(k_z z+m\phi)+i(m-n)\phi_k}$$

$$=2\pi(-i)^n J_n(k_{\rho q}\rho)e^{-i(k_z z+n\phi)}\ . \tag{A6}$$

By introducing the circular cylindrical vector wave functions (16a)–(16c) and recalling the complete property of this set of functions, we have good reason to assume the magnetic field in composite chiral-ferrite media can be represented in form of (15). Then, comparing the coordinate components of (15) with those of (A3) where $P_n(k_z)$, $Q_n(k_z)$, and $R_n(k_z)$ are determined by (A4)–(A6), we derive a set of equations

$$A_q^h(k_z)=-\frac{2i}{k_{\rho q}}B_q(k_z)\ , \tag{A7a}$$

$$\frac{k_z}{k_q}B_q^h(k_z)+iC_q^h(k_z)=-\frac{2}{k_{\rho q}}A_q(k_z)\ , \tag{A7b}$$

and

$$\frac{k_{\rho q}^2}{k_q}B_q^h(k_z)-ik_zC_q^h(k_z)=2\ . \tag{A7c}$$

The solutions to this system of linear equations listed as (A7a)–(A7c) give rise to the expressions of (17a)–(19c).

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